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Erratum

Erratum to “The computational power of Benenson automata” [Theoret. Comput. Sci. 344 (2005) 279–297]

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(i)

We erroneously claim that [Lemma 5.1](#) applies to both non-deterministic and deterministic Benenson automata, while it applies only to deterministic ones. The following corrected lemma holds for both deterministic and non-deterministic Benenson automata. It also exponentially improves on the dependence on D for both for circuit depth and size compared to the paper version. Note that compared with the paper version, the circuit depth has an extra logarithmic dependence on n , and the circuit size has an extra linear dependence on n . Theorem 3.2 follows from [Lemma 5.1](#)(corrected) as before, by taking $D = O(1)$, $S = O(\log n)$, and $L = \text{poly}(n)$.

Lemma 5.1 (Corrected). A function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ computed, possibly non-deterministically, by a Benenson automaton $(S, D, L, \Sigma, n, \sigma, \mathcal{R})$ can be computed by a $O(\log(L/D) \log D + \log^2 D + \log n)$ depth, $O(LD^2 \log D + LDn)$ size circuit.

The proof of [Lemma 5.1](#)(corrected) proceeds as in the paper with the following exceptions. In contrast to what is stated in the paper, when the given Benenson automaton is non-deterministic, in order to generate the initial series of matrices $T_{1,2}(x), \dots, T_{Q-1,Q}(x)$, each gadget A_q may require more than $2D$ bits of input. Indeed, whether any cutting rule applies at the exposed sticky end may depend on all n inputs. In matrix $T_{q,q'}(x)$ the bit at row j , column h (both 0-indexed) is 1 iff $\sigma[(q-1)D+j] \rightarrow_x^* \sigma[(q'-1)D+h]$.² For non-deterministic Benenson automata we can construct the initial matrices $T_{1,2}(x), \dots, T_{Q-1,Q}(x)$ as follows. In each gadget A_q , first we construct a $D \times D$ binary matrix M_q which captures one-step cuts only: the bit at row j , column h (both 0-indexed) is 1 iff $\sigma[(q-1)D+j] \rightarrow_x \sigma[(q-1)D+h]$. To compute a bit of M_q requires a $O(\log n)$ -depth and $O(n)$ -size tree of ORs since there could be $O(n)$ relevant cutting rules for non-deterministic automata. Now note that using matrix multiplication where $+$ is logical OR and \cdot is logical AND, and I is the identity matrix, $(I + M_q)^t$ is the reachability matrix after up to t cuts. Thus $T_{q,q+1} = (I + M_q)^{2D}$ since at most $2D$ cuts are possible to get from the beginning of one segment to the end of the following one. We can compute $(I + M_q)^{2D}$ by squaring $O(\log D)$ times using the same construction as for gadgets B , each of which has depth $O(\log D)$ and size $O(D^3)$. This results in gadgets A of depth $O(\log^2 D + \log n)$ and size $O(D^3 \log D + D^2 n)$. With the rest of the construction as in the paper, the total circuit depth is $O(\log(L/D) \log D + \log^2 D + \log n)$ and size $O(LD^2 \log D + LDn + LD^2) = O(LD^2 \log D + LDn)$ as desired.

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E-mail addresses: dsolov@u.washington.edu (D. Soloveichik), winfree@caltech.edu (E. Winfree).¹ Tel.: +1 626 395 6246; fax: +1 626 584 0630.² There is a typographical error on page 294 in the expression defining $T_{q,q'}$, neglecting to subtract 1 from q, q' . The correct expression is as given here.

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(ii)

Section 5 makes two claims which should not be interpreted more broadly than is justified by the proofs in the paper; specifically: “. . .allowing non-determinism does not increase the computational power. . .”, and “. . .increasing the sticky end size to be larger than $O(\log n)$ does not increase computational power”. The correct sense is as expressed in Section 3: Benenson automata with sticky end size $S = O(\log n)$, cutting range $D = O(1)$, and state string size $L = \text{poly}(n)$ compute the same class of families of functions as $O(\log n)$ -depth circuits, and allowing non-determinism or larger sticky ends does not expand this class. However, it may be possible that non-determinism or larger sticky ends add a lot of computational power if we allow a cutting range $D = O(n)$, for example.

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